CHAOS CONTROL OF GYROSTAT SATELLITE BASED ON THE OPTIMIZATION OF LINEARIZED POINCARE' MAP

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Abstract—In this work, the Dynamical model of the Gyrostat Satellite (GS) is derived using the Euler equation based on the quaternion parameters. Chaos in the system is studied using the time series responses, state space phase trajectories, and the Poincare' section of the system. Also, the Lyapunov exponent criterion is used to prove chaos numerically in the GS system under the effects of the reaction wheels. In order to suppress the chaos in the GS system, a hybrid control strategy is designed based on the linearization of the Poincare' map. For this purpose, Poincare' map of the open-loop system is approximated using the Support Vector Machine technique. The DLQR system is then applied on the linearized Poincare' map for the design of the optimal. The responses of the close-loop system indicate the appropriate performance of the control system along with reducing the control effort due to the properties of the chaotic system.

Keywords—Chaotic Dynamics, Gyrostat Satellite, Chaos Control, Support Vector Machine, Optimal Control

I. INTRODUCTION

The chaotic responses create the irregularity in the GS behaviors, with confusion in the precise satellite mission. Therefore, an analysis of chaos and a chaos control system seem to be necessary for attitude stabilization of the GS. One of the famous chaos control method is the linearization of the Poincare' map introduced by Ott-Grebogi-Yorke called OGY technique [1-3].

The main problem of this control system is the derivation of the related Poincare' map. Soft computing techniques such as Artificial Neural Network and Fuzzy clustering were used for the estimation of Poincare’ map. Support Vector Machines (SVMs) can be applied for the approximation of Poincare’ map in the OGY algorithm. Among the advantages of the SVM is that less adjustable parameters are present, leading to an increase in the learning velocity of the estimated model. In addition, SVMs are not sensitive to changes in the initial conditions, making it possible to use it in applications of the chaotic systems [4,5].

In this work, the dynamical model of the GS system is first derived using the Euler equation under the gravity gradient perturbation torques. In order to explain the kinematic attitude of the GS, the quaternion parameters are used instead of the Euler angles. Then the linear model of the Poincare' map is derived, and the optimal controller on the basis of the Discrete-time Linear Quadratic Regulator (DLQR) is designed on the linearized Poincare’ map. Therefore, the controller is extended for chaos suppression.

II. DYNAMICAL MODEL OF THE SYSTEM

The Gyrostat Satellite involves a rigid main body and three gyros as reaction wheels. The rotation of the gyros is used in the stabilization of the GS for control and stability purposes. Since the gyros are located along the principal axes, the total inertia tensor of the whole GS is diagonal. The orbital motion of the GS is also assumed on the equatorial circular orbit. In order to analysis the dynamics of the GS, three right-oriented orthogonal coordinate systems are used. According to Fig. 1, the coordinate system consists of inertial coordinate system, the orbital frame, and the body coordinates.

Fig 1. The construction of the Gyrostat Satellite
The kinematic model of the satellite is derived using the quaternion representation of the attitude. The quaternion indicates the orientation between the body frame and orbital coordinate instead of Euler angles. The quaternion parameters avoid the singularity problem in the modeling of the system. The quaternion-based kinematic model is expressed as follows.

\[
\begin{bmatrix}
q_1 \\
q_2 \\
q_3
\end{bmatrix} = \begin{bmatrix}
0 & \omega_3 & -\omega_2 & \omega_1 \\
-\omega_3 & 0 & \omega_1 & -\omega_2 \\
\omega_2 & -\omega_1 & 0 & \omega_3 \\
\omega_1 & \omega_2 & -\omega_3 & 0
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2 \\
q_3
\end{bmatrix}
\]

The kinetic model of the GS is derived using the rotational Euler equation based on the calculation of the total angular momentum of the gyrostat satellite as follow [6].

\[
\dot{q}_i = \frac{1}{2} \left[ \begin{array}{c}
\omega_i \\
\omega_j \\
\omega_k
\end{array} \right] \times \left[ \begin{array}{c}
q_1 \\
q_2 \\
q_3
\end{array} \right] + \omega i \left( I_i + 2 \omega \times I \right) + N_C
\]

where \( \hat{N}_C \); \((i : x, y, z)\) are the components of the control stabilizer torques due to the rotation of the reaction wheels, calculated as the derivative of the moment of momentum components of the reaction wheels as

\[
\hat{N}_C = -I_w \left( \omega_x \hat{\Omega}_x + (\omega_y + \hat{\Omega}_y) \hat{j} + (\omega_z + \hat{\Omega}_z) \hat{k} \right)
\]

III. NONLINEAR ANALYSIS OF THE SYSTEM

The equations of motion of the GS on the basis of equations (1-5) are simulated numerically using the Range-Kutta method under the gravity gradient perturbation. The variation of the control parameter along with the effects of the small gravity gradient torques perturb the phase space trajectories of the quasi-periodic system. This leads to the formation of a strange attractor and chaos in the responses of the GS system. The strange attractor and chaotic responses in the GS under the gravity gradient perturbation is illustrated in Fig. 2.

According to Figs. 2, the chaotic responses of the system can be expressed using the stretching, compressing, and folding the orbits in the phase trajectories. On the basis of the Devaney definition of chaos, the chaotic behavior of the system is demonstrated due to the congestion of the Poincare’ section of the system. In addition, conforming to the Fradkov definition of chaos, the basin of attraction for the open-loop model of the GS system as shown in Fig. 2 is chaotic because the orbits are globally bounded and locally unstable based on the Lyapunov stability criterion. Also, based on the Kolmogorov-Arnold-Moser (KAM) theorem, when the perturbation torques such as gravity gradient perturbs the Hamiltonian integrable GS system, the phase portrait of the system can be occupied by the chaotic trajectories [7].
According to the equations of motion of the system, intersection of the trajectories with Poincare' section leads to the generation of discrete points. Poincare' map of the system is derived using the approximation of function from among these points. Based on 1000 points on the Poincare' section, the first 500 points is referred to the network learning and the others points are allocated to the validation (test) of the model. For instance, in the design of the $\Omega_x$-controller with constant values of the $\Omega_x$, the Poincare' map relative to the iterated points in the Poincare' section of $q_1$-$q_2$ is obtained as Fig. 2 given in following.

$$q_1(k + 1) = P_1(q_1(k), q_2(k), u)$$

$$q_2(k + 1) = P_2(q_1(k), q_2(k), u)$$

Using the RBF kernel in the LSSVM structure, we have

$$P_1 = \sum_{i=1}^{N_{spo}} \hat{\alpha}_i \exp(-\frac{(q_1(k)-q_1(i))^2}{\sigma_1^2}) + (q_2(k)-q_2(i))^2$$

$$P_2 = \sum_{j=1}^{N_{spo}} \hat{\alpha}_j \exp(-\frac{(q_1(k)-q_1(j))^2}{\sigma_j^2}) + (q_2(k)-q_2(j))^2$$

To design the $\Omega_x$ and $\Omega_c$-controllers, Poincare' map of the system relative to the Poincare' section of $q_1$-$q_2$ are estimated with constant values of the $\Omega_x$ and $\Omega_c$, respectively.

After estimating the Poincare' map using the LSSVM technique, the linearization of Poincare' map around the stable fixed point is needed. In order to consider the effects of the control input $u$ in the dynamics of the Poincare' map, the $\Omega$ value is varied slightly. Then, the related Poincare' section and Poincare' map are determined, and the control input matrix is obtained using the calculation of the differences rate of the Poincare' maps corresponding to slight variations of $\Omega$. As a result, the linear model of the Poincare' map based on Eq. (12) for the $\Omega_c$-controller is as follows. This process is repeated for the $\Omega_x$ and $\Omega_c$-controllers.

$$A = \begin{bmatrix} \frac{\partial P_1}{\partial q_1(k)} & \frac{\partial P_1}{\partial q_2(k)} \\ \frac{\partial P_2}{\partial q_1(k)} & \frac{\partial P_2}{\partial q_2(k)} \end{bmatrix} = \begin{bmatrix} -0.12 & 0.57 \\ 0.63 & -1.28 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial P_1}{\partial u} \\ \frac{\partial P_2}{\partial u} \end{bmatrix} = \begin{bmatrix} -1.38 \\ 1.47 \end{bmatrix}$$

In order to design the feedback control system, the optimal linear controller on the basis of the Discrete-time LQR method is applied on the system. Therefore,
the DLQR feedback controller is expressed as to minimize the performance index

\[ J(x, u) = \frac{1}{2} \sum_{i=1}^{n}(x_i^T Q x_i + u_i^T R u_i) \]  

(10)

Finally, the control input is designed based on Eqs. (23-26) as

\[ u_i = \begin{cases} 
0.68q_1 + 0.43q_2, & \text{for } \|(q_1, q_2)\| \leq \Delta \\
0, & \text{otherwise} 
\end{cases} \]  

(11)

Simulation results of the attitude controller based on DLQR on the Poincare map are demonstrated in Figs. (4-6). According to Figs. 4 and 5, the quaternion along with the angular velocity responses of the GS system illustrate the proper performance of the control system with regard to the stabilization and suppression of chaos. The system control input corresponding to the angular velocity of the reaction wheels is also demonstrated in Fig. 6.

Fig 4. Time series responses of the quaternions in the control system

Fig 5. Time series responses of the angular velocities in the control system

Fig 6. Time series responses of the control torques

CONCLUSION

In this paper, the attitude model of the GS is derived using the Euler equation with the quaternion parameters. Chaotic responses in the system is analyzed using the time series responses, state space phase trajectories, and the Poincare’ section of the system. Also, the Lyapunov exponent criterion is used to prove chaos numerically in the GS system under the effects of the reaction wheels. In order to control of chaos in the GS system, a hybrid controller is designed based on the linearization of the Poincare’ map. Therefore, the optimal DLQR is applied on the Poincare’ map. The Poincare’ map of the open-loop system is approximated using the LSSVM technique. The DLQR system is then applied on the linearized Poincare’ The responses of the close-loop system indicate the appropriate performance of the control system along with reducing the control effort due to the properties of the chaotic system

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