FRACTIONAL ORDER FILTER BASED ON FRACTIONAL CAPACITORS AND FRACTIONAL INDUCTOR

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Abstract- This paper presents a new approach to design a fractional order filter based on the concepts of fractional order capacitors and fractional order inductors. The filters are constructed using a fractional order inductance of order $\alpha$ ($0 < \alpha < 2$) and fractional order capacitors (FC) of order $\beta$ ($0 < \beta < 1$). The fractional order inductances are realized through generalized impedance converter (GIC) circuits where FC is used as a component in it. The authors have shown by simulating GICs used in fractional order filter, that fractional order inductance can be realized, in general, through GIC and a fractional order capacitors. Finally, the performance of the fractional order tuned filter has been studied.

Key Words - Filter, Fractional capacitor, Fractional inductor, GIC.

I. INTRODUCTION

The transfer function of a fractional order filter comprises of real powers of ‘$s$’ in numerator and denominator. This is in contrary to integer order filters where all the powers of ‘$s$’ in the transfer function are integers. Traditional continuous type analog filters are of integer order, since their transfer functions contain integer powers of ‘$s$’ But, if one or more conventional capacitors in the circuits are replaced by fractional capacitor, the filter transfer function will contain real powers of ‘$s$’. This is because; the impedance of fractional capacitor (FC) is expressed as $Z = \frac{1}{Cs^\alpha}$ where $C$ is the capacitance of the FC and $\alpha$ ($0 < \alpha < 1$) is its order. Moreover, by using general impedance converter (GIC), it is also possible to construct a fractional inductor; order of fractional inductor and capacitor can also be raised to $\alpha$ ($0 < \alpha < 2$). It is thus expected that the generalization of transfer function of the filters with real power of ‘$s$’ will enhance the flexibility of the design and the improvements of the performance.

With the advent of fractional-order devices, there has been a growing interest among researchers to examine the performance of fractional-order filters as these filters have wide applications in the fields of signal processing, controller design, non linear system identification etc. The fractional order filters can also be used in many fields of science and engineering for modeling of music instruments, speech coding and synthesis, comb filter design and analog digital conversion, scale conversion and linear prediction, etc. [1-7]. The design and realization of fractional order
filter are done using recently developed two terminal fractional order capacitors [8, 9, 10, 11]. However, fractional order low frequency filter are generally realized with the fractional order capacitors of order greater than one to get higher stop band attenuation as discussed in section 4. To realize a fractional order capacitor of order \( \alpha \) greater than one (1 < \( \alpha \) < 2), generalized impedance converter (GIC) can be used. Also, ladder filters require large inductors if designed for low frequencies. These inductors are not only inconvenient; their resistance can also have an adverse effect on the frequency response of the filter [9, 10, and 11]. Similarly realization of fractional order filters where fractional order inductor is the key component, are limited so far due to non availability of a two terminal fractional order inductor. Here, to realize a fractional order inductor, a two terminal fractional order capacitor is used in a generalized impedance converter (GIC). This paper is organized as follows: Sect.2 presents the background of fractional order capacitor and Inductor. Sect.3 presents the GIC as fractional capacitor and fractional Inductors. Application of GIC as fractional order filter and there after its analysis has been presented in Section 4. Similarly concluding remarks are summarized in Sect.5.

II. THEORETICAL BACKGROUND OF FRACTIONAL ORDER CAPACITOR AND INDUCTOR:

Fractional Capacitor (FC):
A passive circuit element that gives phase angle between 0 to -90 degree and remains constant with frequency is called a fractional capacitor (FC) [6]. Here \( CF \) is called the fractional inductance and \( \alpha \) is its order. The magnitude characteristics of an ideal fractional inductor are a straight line with slope -20\( \alpha \) dB/decade while the phase angle remains constant for all frequencies as shown in figure.

\[
\mathbf{Z} = \frac{1}{CF \cdot s^\alpha} = \frac{1}{CF \cdot \omega^{\alpha}} \angle -\frac{\pi \alpha}{2} \quad (1)
\]

Here \( CF \) is the fractional capacitance of the FC and \( \alpha \) is its order. The unit of fractional capacitance is expressed as \( F \cdot s^{1-\alpha} \) where; \( s \) denotes time in second and \( F \) denotes farad [18]. Similarly, the magnitude characteristics of an ideal FC are a straight line with slope -20\( \alpha \) dB/decade while the phase angle remains constant for all frequencies as shown in figure.

\[
\mathbf{Z}(j\omega) \quad \text{Fig 1(a). Magnitude plot of FC}
\]

\[
\angle \mathbf{Z}(j\omega) \quad \text{Fig 1(b). Phase plot of FC}
\]

Fractional Inductor (FI):
A passive circuit element that gives constant phase angle between 0 to +90 degree and remains constant with frequency, is called a fractional inductor [38]. The impedance of a fractional inductor is expressed by

\[
\mathbf{Z} = \frac{1}{LF \cdot s^\alpha} = \frac{1}{LF \cdot \omega^{\alpha}} \angle \frac{\pi \alpha}{2} \quad (2)
\]

dB/decade while the phase angle remains constant for all frequencies. So far, the realization of fractional inductor similar to a single component fractional
capacitor is not available in literature. However, the simulation of grounded fractional inductor using general impedance converter (GIC) network is discussed in this paper.

Similarly, Z1, Z3 are selected as impedances of two capacitance i.e.,

\[ Z_1 = \frac{1}{C_2 s^{\alpha}} \]

However, Z4 is selected as the impedance of a fractional order capacitor i.e., \( \frac{1}{C_2 s^{\frac{\alpha}{2}}} \). With the above parameter, the impedance of the GIC can be found as

\[ Z = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} = \frac{1}{C_2 s^{\frac{\alpha}{2}}} = \frac{1}{C_2 s^{\theta}} \]  

This implies that the impedance Z of GIC is the impedance of a FC [9, 10] whose order is \( b \) i.e., \( b = 2 - g \).

Case 2: To simulate fractional order inductor Z4 is selected as the impedance of a fractional order capacitor i.e., \( \frac{1}{C_2 s^{\frac{\alpha}{2}}} \), and all other impedances are selected as resistors of value R. With the above parameter, the impedance of the GIC can be found as

\[ Z = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} = \frac{1}{C_2 s^{\theta}} \]

This implies that the impedance Z of GIC is the impedance of a fractional order inductor [13] whose order is \( g \).

IV. GIC IN FRACTIONAL ORDER FILTER

A GIC can be used in parallel LFCF circuit where the parallel LFCF circuit is used as a narrow bandpass filter circuit (NBF). A NBF is basically used as a fractional order tuned harmonic filter, tuned to a single harmonics frequency with a low harmonic impedance characteristic. These filters are used to provide an alternative impedance path for harmonic currents generated by the nonlinear load. Hence, these filters are much desirable in power system engineering for reducing harmonic voltage and current distortion through alternate circuit path operation.
The theory and implementation of advanced techniques such as harmonic current injection, dc ripple injection, and series/shunt active filter systems have been studied in [14, 15, 16] for compensating harmonic voltages and currents. However, we have proposed a fractional order filter tuned to a particular frequency, is based on fractional calculus theory. It is simple in structure, convenient maintenance and lower price. Tuned frequency of the filter is a function of the order of fractional inductance and capacitance which add an extra design freedom. Basically it consists of a resistor, a fractional inductor (which is simulated through GIC) and fractional capacitors, where the fractional order capacitor can be connected in parallel with fractional order inductor shown in figure 1. It is said to be tuned on that frequency where the fractional tuned circuits offers minimum impedance.

Here, GIC is simulated to a fractional inductor of order $\alpha$. and fractional capacitance of order $\beta$. Though general notation $\tilde{\alpha}$ is taken previously as the order of fractional capacitor and fractional inductor.

$$I(s) \over V(s) = Y(s) = \frac{(RC_F s^\alpha + L_F C_F s^{\alpha - \beta})}{(RC_F s^\beta + L_F C_F s^{\alpha + \beta}) + 1} \tag{7}$$

Where, the impedance of fractional order inductor is $LF \alpha$, and the impedance of fractional capacitor is $\frac{1}{C_F s^\alpha}$.

Using $s^\alpha = \omega_0^\alpha e^{j\alpha}$ and $s^\beta = \omega_0^\beta e^{j\beta}$, equation (5) becomes

$$Z_p(j\omega) = K\left(\frac{R^2 + L_F^2 \omega_0^{2\alpha} + p^2 \omega_0^{2\beta}}{\omega_0^{2\alpha} + \frac{R^2}{L_F^2} \omega_0^{2\beta} + \frac{2p}{L_F} \omega_0^{\alpha + \beta} + \frac{\omega_0^{2\alpha} + \frac{R}{L_F} \omega_0^{\alpha + \beta}}{L_F} \right) \tag{8}$$

Where, $p = 2RL_F \cos(\frac{\pi \omega_0}{2})$, $q = \frac{2R \cos(\pi \beta)}{C_F}$, and

$$r = \frac{2L_F \cos(\frac{\pi (\alpha + \beta)}{2})}{C_F}$$

The admittances (the reciprocal of the impedances as given in equation (5)), are simulated as shown in figure 2. Here, the term $\sqrt{LF CF/R}$ (quality factor of integer order filter) has been maintained constant for both fractional order and integer order tuned filter.

As in figure 2, sharper tuning can be achieved when inductor or capacitors are replaced with their fractional order counterpart. Based on structure as shown in figure 4, the fractional order filter is simulated in MATLAB. The GIC is simulated as a fractional inductor of order 1.8. To achieve, parameter $Z_2$ is selected as capacitor, $Z_4$ as fractional capacitor of order 0.8 and all other parameters are taken as resistors.
When $\alpha=1$ and $\beta=1$, it becomes an integer order filter. The simulation result is shown in figure 5.

When $\alpha=1.8$ and $\beta=0.2$, the simulation results are shown in figure 6. From impedance characteristics of integer order filter, its graphics are consistent with the simulation results as in [15]. The impedance characteristics of fractional order filter (figure 6) shows that the low impedance is obtained at tuned frequency but high impedance at other frequency. Hence, it will increase greatly the filtering characteristics and reduce power loss.

VI. CONCLUSIONS

In this paper the use of generalized impedance converter is used to simulate both fractional order inductor (FI) and fractional order capacitor (FC). The concept of FC and FI has also been noted and the basic concept has also been analyzed. It has been seen that by using GIC, the fabrication of fractional order filter can be possible. It has been observed that due to the presence of two parameter order $\alpha$ and $\beta$, additional degrees of design freedom are available for better flexibility of filter design.

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