EMPIRICAL STUDY ON THE MARKOV-MODULATED REGIME-SWITCHING MODEL WHEN THE REGIME SWITCHING RISK IS PRICED

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Abstract—In the current literature, Markov regime-switching option models are often developed and tested by using purposely designed specimen/artificial data, rather than real market financial data. In this paper, we investigate the option valuation model empirically, which is deemed to price the regime-switching risk of an economy using a hidden Markov regime-switching model with the risky underlying asset being modulated by a discrete-time, finite-state, hidden Markov chain whose states represent the hidden states of an economy. We apply such a model on the pricing of Hang Seng Index options based on the real-world financial data from Oct. 2009 to Oct. 2010. We discuss several aspects of the application of the adopted model, and conclude that the model still suffers from the assumption of Geometric Brownian Motion of the underlying asset in regimes. This research highlights the fundamental issues with regard to the current development of the Markov-modulated regime-switching option models.

Keywords—Option pricing, Regime-switching, Hidden Markov model, Esscher transform, HSI index options

I. INTRODUCTION

In recent years, the option valuation problem under regime-switching has received considerable interest in literature. A key feature of regime-switching models is that model parameters are modulated by a Markov chain whose states represent states of business cycles (See Hamilton (1989)). Some early works on option pricing under regime-switching conditions include Naik (1993), Guo (2001), Buffington and Elliott (2002), Elliott et al. (2005), Siu (2008) and others. To be more specific, Guo (2001) investigated an option pricing problem in an incomplete market modelled by adjoining the Geometric Brownian Motion (GBM) for stock returns with a Markov chain in a Black-Scholes (1973) economy. Buffington and Elliott (2002) considered the option pricing problems for European and American options in a Black-Scholes market in which the states of the economy are described by a continuous-time, finite-state, Markov chain. Yao et al. (2003) investigated the pricing of European options under a Markov-modulated GBM and determined an equivalent martingale pricing measure for the Markov-modulated GBM. Elliott et al. (2005) proposed the use of a regime-switching version of the Esscher transform to determine an equivalent martingale measure for valuing options in a Markov-modulated Black-Scholes-Merton economy. Indeed,
Gerber and Shiu (1994) pioneered the use of the Esscher transform in finance, in particular in option valuation. It provides a convenient method to specify an equivalent martingale measure. Siu (2008) justified the use of the Esscher transform for option valuation in a regime-switching diffusion model and a regime-switching jump-diffusion model using a game theoretic approach. Siu and Yang (2009) considered a modified version of the Esscher transform used in Elliott et al. (2005) to incorporate explicitly the intensity matrix of the Markov chain in the specification of an equivalent martingale measure. Siu (2011) demonstrated, through a rigorous mathematical proof, that an optimal equivalent martingale measure selected by minimizing the relative entropy between an equivalent martingale measure and the real-world probability measure does not price the regime-switching risk. Elliott et al. (2013) investigated the pricing of both European and American-style options when the price dynamics of the underlying risky assets are governed by a Markov-modulated constant elasticity of variance process. So far, several extended regime-switching models (such as with a jump-diffusion process) have been developed to price different types of options, for example, currency options (Bo, et al., 2010), bond options (Shen, et al., 2013), foreign equity options (Fan, et al., 2014; Lian, et al., 2016), and others.

In terms of option valuation principles, it has been established (see, Harrison and Kreps (1979) and Harrison and Pliska (1981)) that the absence of arbitrage is equivalent to the existence of an equivalent martingale measure under which all discounted price processes are martingales. However, when the market is incomplete, there are more than one equivalent martingale measures. How to choose a consistent pricing measure from the set of equivalent martingale measures becomes an important problem. As one of the most important approaches, the minimal relative entropy approach is often employed to select an equivalent martingale measure from its canonical space. As discussed in Siu (2011), Miyahara (1996) was the first to introduce the minimal entropy martingale measure (MEMM) approach to select an equivalent martingale measure in an incomplete market. Nowadays, the MEMM approach has become one of the major approaches for option valuation in an incomplete market. The basic idea of the MEMM approach is to select an equivalent martingale measure so as to minimize the “distance” between an equivalent martingale measure and a real-world probability measure described by their relative entropy. Consequently, the MEMM is the equivalent martingale measure which is closest to the real-world probability measure. For details about the MEMM approach for option valuation, interested readers may refer to works by Miyahara (2001), and Fujiwara and Miyahara (2003).

In this paper, we conduct the empirical studies on the pricing of Hang Seng Index Options (HSI) by the Esscher transform and MEMM approaches. We model the price dynamics of the underlying risky asset which are governed by a Markov-modulated geometric Brownian motion using a novel model that was proposed by Siu et al. (2009), in which the regime-switching risk was supposed to be priced. We suppose that the drift and the volatility of the underlying asset are modulated by an observable continuous-time, finite-state Markov chain, whose
states represent observable states of an economy. Unlike most of the previous works of model development, we pay more attention to the option pricing performance of the model. In the current literature, regime-switching option models are often developed and tested by using purposely designed specimen/artificial data, rather than real market financial data. Our research is important in terms of calibrating a theoretical Markov regime-switching model against a real financial problem, and assessing the actual pricing ability of the theoretical model. This kind of research work is urgently needed in the current development and testing of various Markov-modulated regime-switching option models.

The rest of the paper is organized as follows. The next section describes the model dynamics. In section three, we present the two-stage pricing method. In section four, we present the numerical examples for the computation of the option prices. We also present and discuss the results of numerical experiments. The final section concludes the paper.

II. THE OPTION MODEL UNDER INVESTIGATION

The option pricing model being investigated is the one that was proposed by Siu et al. (2009), which is deemed to have the advantages of pricing the regime-switching risk for an option over other similar models under the framework of a general Markov-modulated regime-switching of an economy. The main feature of the model is that the pricing of an option is conditional on a single initial state of an economy rather than a whole path of the price dynamics of the underlying asset of the option. We shall give a brief introduction of the model being studied in this section as follows.

A. The Price Dynamics

The main goal in this section is to introduce the price dynamics which is dominated by a Markov-modulated geometric Brownian motion. Such a framework has been well documented in Elliott (1993), Elliott et al (1994), and Siu and Yang (2009). Consider the money account B and stock S in a financial model, we shall describe the price dynamics of these two assets. Firstly, we define the hidden Markov chain \{X_t\}_{t \in T} on the complete probability space \((\Omega, F, P)\) with a finite \((x_1, x_2, \ldots, x_N)\), where \(T\) denotes the finite time horizon \([0,T]\) and \(P\) denotes a real world probability measure. According to Elliott et al. (1994), the state space of \{X_t\}_{t \in T} is defined by a finite set of unit vectors \(e := \{e_1, e_2, \ldots, e_N\}\). Where \(e_i = (0, 1, \ldots, 0) \in \mathbb{R}^N\).

Then, Elliott (1993) and Elliott et al (1994) provide the following semi-martingale decomposition for \{X_t\}_{t \in T}:

\[
X_t = X_0 + \int_0^t QX_s ds + M_t .
\] (1)

Where \(Q\) denotes rate matrix \([q_{ij}(t)]_{i,j=1,2,\ldots,N}\) and \(\{M_t\}\) is \(\mathbb{R}^N\)-valued martingale with respect to the filtration which generated by \{X_t\}_{t \in T} and the measure \(P\).

Assume that \{r_t\}_{t \in T} denotes the market interest rate of the money market account at time \(t\). We suppose that

\[
\tau_t := \tau(t, X_t) = (r_t, X_t) .
\] (2)

Where \(r := (r_1, r_2, \ldots, r_N) \in \mathbb{R}^N\) with \(r_l > 0\) for each \(l = 1, 2, \ldots, N\).

Therefore, the price dynamic of money market account \{B_t\}_{t \in T} is modeled by

\[
B_t = e^{-r_t} = \exp \left(-\int_0^t r_u du \right) .
\] (3)
In addition, assume that \{\mu_t\}_{t \in \mathbb{T}} and \{\sigma_t\}_{t \in \mathbb{T}} are the appreciation rate and the volatility of stock \(S\) respectively, which are defined as follows:

\[
\mu_t := \mu(t, X_t) = \langle \mu, X_t \rangle,
\]

\[
\sigma_t := \sigma(t, X_t) = \langle \sigma, X_t \rangle,
\]

Where \(\mu := (\mu_1, \mu_2, \ldots, \mu_n) \in \mathbb{R}^n\) and \(\sigma := (\sigma_1, \sigma_2, \ldots, \sigma_n) \in \mathbb{R}^n\).

Then, we use the Markov-modulated geometric Brownian motion with jump to define the dynamic of underlying stock \(\{S_t\}_{t \in \mathbb{T}}:\)

\[
dS_t = \mu S_t dt + \sigma S_t dW_t, \quad S_0 = s,
\]

Where \(\{W_t\}_{t \in \mathbb{T}}\) denote the standard Brownian motion on \((\Omega, \mathcal{F}, P)\).

Then the price dynamic of \(S\) can be written as

\[
S_t = S_s \exp(Y_t),
\]

where \(Y_t\) denotes the logarithmic return of \(S\) over the interval \([0, t]\), and

\[
Y_t = \int_0^t (\mu_s - \frac{1}{2} \sigma^2_s) ds + \int_0^t \sigma_s dW_s .
\]

### B. Option Pricing under Regime Switching

In this section, the regime-switching Esscher Transform and risk-neutral Esscher parameters will be described. Let

\(\mathcal{G}_T\) be the \(\sigma\)–algebra \(\mathcal{F}_T^X \vee \mathcal{F}_T^X\) which is generated by \(\{X_t\}_{t \in \mathbb{T}}\) and \(\{S_t\}_{t \in \mathbb{T}}\):

under the \(P\)-argumentation of natural filtrations.

Moreover, let \(\theta_t\) be the regime switching Esscher parameter at time \(t\), which can be written as follows:

\[
\theta_t := \theta(t, X(t)) = \langle \theta, X_t \rangle,
\]

Where \(\theta := (\theta_1, \theta_2, \ldots, \theta_n) \in \mathbb{R}^n\).

Following Elliott (1982), write

\[
(\theta \cdot Y)_t = \int_0^t \theta(u)dY(u) \quad \text{for each } t \in T.
\]

Then we define the regime-switching Esscher Transform on \(Q_\theta \sim \mathcal{P}\) on \(\mathcal{G}_T\) as follows:

\[
\frac{dQ}{dP} := e^{\langle \theta, Y \rangle_T} = \Lambda_T .
\]

Where \(E[\cdot]\) denotes an expectation under \(P\).

Then we consider the European option with payoff \(V(S_T)\) at maturity \(T\). Therefore, the conditional price of option given \(\mathcal{G}_T\) is:

\[
V_t := E^P \left[ \exp \left( -\int_0^T r_u du \right) V(S_T) \bigg| \mathcal{G}_T \right] (13)
\]

When the \(S_t = s\) and \(X_t = x\) the value of option is:

\[
V(t, s, x) = E^P \left[ \exp \left( -\int_0^T r_u du \right) V(S_T) | S_0 = s, X_0 = x \right] (14)
\]

For a European call option, it can be evaluated as follow according to (14), i.e.

\[
C(0, S_0, X_0) = E^P \left[ \exp \left( -\int_0^T r_u du \right) (S_T - K)^+ | S_0, X_0 \right] (15)
\]

The function can be re-written as follow by using regime-switching Esscher Transform as proposed in Siu et al. (2009):

\[
C(0, S_0, X_0) = E^P \left[ \frac{dQ}{dP} \exp \left( -\int_0^T r_u du \right) (S_T - K)^+ | S_0, X_0 \right] (16)
\]

We will use Monte Carlo simulations to estimate the call option prices. First, we divided the time horizon \([0, T]\) into \(N\) subintervals \([t_j, t_{j+1}]\) \((j = 0, 1, \ldots, J - 1)\) with equal length \(\Delta t = \frac{T}{J}\) where \(t_0 = 0\) and \(t_J = T\). Then, for each \(t = 1, 2, \ldots, L\), simulate the discrete-time version of Markov chain \(X\) and obtain \(\{Y_j(t)\}_{j=1}^L\) and its corresponding \(\{\mu_j(t)\}_{j=1}^L\) and \(\{\sigma_j(t)\}_{j=1}^L\).

Finally, the \(Y_{t+1}^j\) is defined as:

\[
Y_{t+1}^j = Y_t^j + \left( \mu_t^j - \frac{1}{2} \sigma_t^j \right) \Delta t + \sigma_t^j \xi_{t+1}^j (17)
\]

Where \(\xi_{t+1}^j \sim N(0, \Delta t)\).

The parameters in Eqn. (16) can be obtained in practice except for the risk-neutral Esscher parameters \(\theta_t\). Therefore, in the next section we will present the method to calculate \(\theta_t\).
C. Determination of risk-neutral Esscher parameters

First, we need to define a \( \{\mathcal{G}_t, \mathcal{F}_t\} \)–martingale \( \{\Lambda_t\}_{t \in \mathcal{T}} \) as defined in Siu and Yang, (2009), i.e.

\[
\Lambda_t := E[\Lambda_T | \mathcal{G}_t], \quad t \in \mathcal{T}.
\]

Let \( \tilde{S} := e^{-\int_{t_0}^{t} \theta_u dS(u)} \), for each \( t \in \mathcal{T} \). Here, the martingale condition is given by considering an enlarged filtration as follows:

\[
\tilde{S}(u) := E^{Q}[S(t) | \mathcal{G}(u)], \quad \text{for any } t, u \in \mathcal{T}, \quad \text{with } t \geq u,
\]

where \( E^Q \) denotes expectation under \( Q^\theta \).

Define \( \lambda_i(\theta) := \theta \mu_i - \frac{1}{2} \theta \sigma_i^2 + \frac{1}{2} \frac{\theta_i^2}{\sigma_i^2} \) for \( i = 1, 2, \ldots, N \),

\[
\lambda_i(\theta) := -\gamma_i + (\theta_i + 1) \mu_i - \frac{1}{2} (\theta_i + 1)^2 \sigma_i^2 + \frac{1}{2} (\theta_i + 1)^2 \sigma_i^2
\]

and

\[
\lambda(\theta) := (\lambda_1(\theta_1), \lambda_2(\theta_2), \ldots, \lambda_N(\theta_N)) \in \mathbb{R}^N.
\]

Then, the martingale condition is satisfied if and only if

\[
\langle e^{(Q+\text{diag}(\lambda(\theta)))(t-u)} X_{ut}, 1_N \rangle - \langle e^{(Q+\text{diag}(\lambda(\theta)))(t-u)} X_{ut}, 1_N \rangle = 0
\]

for all \( X_u \) and for all \( t, u \in \mathcal{T} \) with \( t \geq u \).

The proof of the martingale condition employs a version of Bayes’ rule and the definition of \( \vee_t \) in Eqn. (18), and may be found in Siu et al. (2009) and Elliott and Osakwe (2006), so we don’t repeat here.

To expand the term \( e^{(Q+\text{diag}(\lambda(\theta)))(t-u)} X_{ut}, 1_N \), we will use the equation \( \exp(M) = \sum_{n=0}^{\infty} \frac{M^n}{n!} \). The first-order approximation may be used to estimate the risk-neutral Esscher parameters \( (\theta_1, \theta_2, \ldots, \theta_N) \) that corresponds to the Esscher parameters generated in the works by Elliott et al. (2005), whilst the second-order approximation may be used to estimate \( (\theta_1, \theta_2, \ldots, \theta_N) \) as well. Siu and Yang (2009) proved that the Esscher parameters can first be evaluated by Eqn. (22) when using the first-order approximation of \( \exp(M) \), i.e.

\[
\theta_i = \frac{\gamma_i - \mu_i}{\sigma_i^2}\frac{1}{\theta_i} \quad \text{for } i = 1, 2, \ldots, N,
\]

In this paper, besides using the first order approximation (named Model I), we will also use the two-order approximation to estimate the risk-neutral Esscher parameters (Named Model II). Consequently, there will be more than one pair of \( (\theta_1, \theta_2) \) in the latter case when the equation (21) is solved for the regime-switching problem of two states. The min-max entropy method will therefore be used to select an optimal pair of \( (\theta_1, \theta_2) \). It is claimed that Model II can price the regime-switching risk of an economy, while Model I cannot (Siu et al. (2009)).

D. Relative entropy for equivalent martingale measure

The concept of entropy plays an important role in mathematical finance. Miyahara (1999) was the first to introduce the minimal entropy martingale measure (MEMM) approach to select an equivalent martingale measure in an incomplete market. Nowadays, the MEMM approach has become one of the major approaches for option valuation in an incomplete market. As we have discussed before, there are more than one set of \( (\theta_1, \theta_2, \ldots, \theta_N) \) satisfying the equation (21). We will choose an optimum set of risk-neutral Esscher parameters by \( (\theta_1, \theta_2, \ldots, \theta_N) \) minimizing the maximum entropy between an equivalent martingale measure and the real world probability measure over different states. The principle of maximum entropy indicate that the probability distribution which best represents the current state of knowledge is the one with largest entropy. To maximize entropy, Siu et al.
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(2009) define the entropy between \( Q_{\theta} \) and \( P \) conditional on \( X_0 \) as below,
\[
I(Q_{\theta}, P|X_0) := E\left[ \frac{dQ_{\theta}}{dP} \ln \left( \frac{dQ_{\theta}}{dP} \right) X_0 \right]
= E\left[ e^{(Q_{\theta} + \lambda_{(\theta)T})T X_0,1_2} \right] - E\left[ e^{(Q + \lambda_{(\theta)T})T X_0,1_2} \right]
\]
Where \( X_0 = e_i, i = 1, 2 \) for state 1 and state 2.

Define the \( I(Q_{\theta}|P) \) is the maximum entropy between \( Q_{\theta} \) and \( P \) over the different values \( X_0 \)
\[
I(Q_{\theta}|P) := \max_{i=1,2,\ldots,N} I(Q_{\theta}, P|X_0 = e_i)
\]
(24)

Note that \( N=2 \) in our research.

Then, a set of risk-neutral Esscher parameters are selected when \( I(Q_{\theta}|P) \) is minimized.

III. NUMERICAL EXPERIMENTS

In the part, we present a real data experiment to investigate the option model outline in the above mentioned sections. We use a data set of daily closing prices of Hang Seng Index (HSI), from 31 Oct 2009 – 31 Oct 2010, which was retrieved from the HK stock exchange. There are in total 252 observations.

Figure 1: Hang Seng index prices between 31 Oct 2009 and 31 Oct 2010

In this investigation, the number of regime states is taken to be two. The estimated Markov regime-switching parameters are
\[
(\hat{\mu}_1, \sigma_1) = (0.0017, 0.0084); \quad \hat{\eta}_1 = 0.007;
(\hat{\mu}_2, \sigma_2) = (-0.0003, 0.0131); \quad \hat{\eta}_2 = 0.007;
\]
The transition probabilities are estimated to be
\[
P = \begin{pmatrix}
0.99 & 0.01 \\
0.05 & 0.95
\end{pmatrix}
\]

We extend the work of Siu et al. (2009) so that the model can deal with the cases when the rate matrix are controlled by two different components, i.e. the rate matrix components can be calculated as follows,
\[
q_{12} = -q_{11} = -\frac{P_{12} \ln(1 - P_{12} - P_{21})}{\Delta(P_{12} + P_{21})}
q_{21} = -q_{22} = -\frac{P_{21} \ln(1 - P_{21} - P_{12})}{\Delta(P_{12} + P_{21})}
\]
(24)

Suppose the current time is \( t_0 \). Without loss of generality, we put \( t_0=0 \) and \( S_0 \) (the index HIS) is 23,652.94 as observed on 1 Nov 2010 on the HK stock exchange.

Firstly, we present the results of selecting optimal martingale measures in Table 1 for some typical cases using the MEMM. Taking the case that \( K=21,000 \) and \( T=0.417 \) as an example, there are three pairs of Esscher \((\bar{\theta}_1, \bar{\theta}_2)\) parameters that satisfy the equivalent martingale conditions Eqn. (21), i.e. \((75.11, 42.54), (-1889.67, 1507.73), \) and \((1968.81, -1500.49)\). By deciding a minimal of the maximum entropies, we can identify an optimal martingale measure. The option prices are then evaluated using the selected Esscher parameters. For the case \( K=21,000 \) and \( T=0.417 \), the option prices obtained by using Model I and Model II, respectively, are
2658.4 (1st order approximation, Model I)
2658.7 (2nd order approximation, Model II).

We have further computed for other index options with different strike prices and maturities, and found that the prices for each option obtained by Model I and Model II are essentially no much difference. So we don’t present the detailed corresponding results
here. Instead, we will conduct further computations of the index options using Model II only.

Table 1 Esscher parameter selection by the MEMM

<table>
<thead>
<tr>
<th>Options</th>
<th>Esscher parameters ($\hat{\theta}_0, \hat{\theta}_1$)</th>
<th>Entropy (H,</th>
<th>Max Entro</th>
<th>Selected ((\hat{\theta}_0, \hat{\theta}_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>K=2</td>
<td>(75.11, 42.54)</td>
<td>(-7.15, 3.14)</td>
<td>1.84</td>
<td>1.84</td>
</tr>
<tr>
<td>T=0.417</td>
<td>(1889.67, 1507.73)</td>
<td>(-6.56, 1.84)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(1968.81, 1500.49)</td>
<td>(-4.04, 2.18)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>K=2</td>
<td>(75.11, 42.54)</td>
<td>(-8.11, 3.14)</td>
<td>3.09</td>
<td>3.09</td>
</tr>
<tr>
<td>T=0.67</td>
<td>(1974.33, 1504.03)</td>
<td>(-4.97, 2.75)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(1895.64, 11.26)</td>
<td>(-4.48, 2.75)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>K=2</td>
<td>(75.11, 42.54)</td>
<td>(-8.74, 3.14)</td>
<td>3.36</td>
<td>3.36</td>
</tr>
<tr>
<td>T=1</td>
<td>(1976.78, 1505.59)</td>
<td>(-5.58, 3.70)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2 Market and estimated HSI option values at Nov 2010 have been evaluated using Model II (i.e. the regime-switching risk is priced) and the results are summarized in Table 2 in together with their market prices. The strike prices range from 20,800 to 24,400, and the maturities of the options are 3 months, 5 months, and 11 months respectively. It can be seen that, firstly, the regime-switching model yields results smaller than the market option prices, especially, for the options in the money. We then plot the option prices in comparisons with their market prices in Figure 1.

Figure 2. Comparisons between estimated option prices (using Model II) with market prices

It can be seen in Figure 2 that general trends of the option prices along with the strike prices and the maturity times seem reasonable, for instance, the index option’s price decreases when the strike price increases in all the cases studied, whilst it increases slightly when the maturity time becomes longer. However, the regime-switching option model being used does not give good predictions of the option

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prices especially for the on-the-money and out-of-the-money options. The finding is consistent with what was found in the works by Liew et al. (2010) that their regime-switching option model could not give good results for certain options such as out-of-the-money options.

To investigate the option pricing problem further, we then plot the estimated option prices given by Model II in comparison with the Black-Scholes (BS) model in Figure 3. The BS option prices are obtained for the two different regimes (state 1 and state 2) of the economy separately. It is shown that the Markov modulated regime-switching model with the regime-switching risk being priced is very consistent with the results given by the BS model. For options with a longer maturity time, the Markov regime-switching model yields results slightly smaller than the corresponding Black-Scholes results. However, for on-the-money and out-of-the-money options, the Markov regime-switching model approaches zero values swiftly the same as the BS model. In this regard, it seems the Markov regime-switching option model studied doesn’t benefit from its ability of pricing the regime-switching risk. This is mainly due to the fact that the assumption of Geometric Brownian Motion still applies to the underlying asset of the options in each of the regimes of the economy in the option model.

IV. CONCLUSIONS

In conclusion, the main purpose of the paper is to conduct an empirical analysis of the real-world index options, namely Hang Seng Index (HSI) options, using the framework of the Markov regime switching model that was proposed by Siu et al. (2009). The price dynamics of the risky underlying asset is modulated by a hidden Markov chain of finite number of states. We show that the option prices obtained by using Model I and Model II are essentially the same for each option, which suggests that the regime-switching risk may not play a key role in the option prices of the HSI options being investigated. We also observed that the prices of the HSI options predicted by the Markov regime-switching model are very comparable to the BS model.
model, and the former doesn’t show distinct advantage over the traditional Black-Sholes model in terms of pricing on-the-money and out-of-the-money options in the case of HSI options being priced. We have also highlighted the current challenges for the Markov Regime-Switching models to price the on-the-money and out-of-the-money options in the real world financial problem.

V. REFERENCES